INTERNATIONAL JOURNAL OF COMPUTER ENGINEERING & TECHNOLOGY (IJCET)

ISSN 0976 – 6367(Print)
ISSN 0976 – 6375(Online)
Volume 5, Issue 4, April (2014), pp. 165-174
© IAEME: www.iaeme.com/ijcet.asp
Journal Impact Factor (2014): 8.5328 (Calculated by GISI)
www.jifactor.com

IJCET

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NEW CHARACTERIZATION OF KERNEL SET IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce a kernelled fuzzy point, boundary kernelled fuzzy point and derived kernelled fuzzy point of a subset A of X, and using these notions to define kernel set of fuzzy topological spaces. Also we introduce fuzzy topological kr- space. The investigation enables us to present some new fuzzy separation axioms between FT_0 and FT_1- spaces.

Keywords: Fuzzy Topological Space, Kernelled Fuzzy Point, Boundary Kernelled Fuzzy Point, Derived Kernelled Fuzzy Point, Kernel Set, Weak Fuzzy Separation Axioms, FR_i- space, $i = 0,1$.

1. INTRODUCTION

The concept of fuzzy set and fuzzy set operations were first introduced by L. A. Zadeh in 1965 [8]. After Zadeh 's introduction of fuzzy sets, Chang [3] defined and studied the notion of fuzzy topological space in 1968. In 1997, fuzzy generalized closed set (Fg - closed set) was introduced by G. Balasubramania and P. Sundaram [6]. In 1998, the notion of Fgs - closed set was defined and investigated by H. Maki el al [7]. In 2002, O. Bedre Ozbakir [11], defined the concept of fuzzy generalized strongly closed set. In 1984, fuzzy separation axioms have been introduced and investigated by A. S. Mashour and others [1].

In this paper we introduce a new characterization of kernel set through our definition kernelled fuzzy point, boundary kernelled fuzzy point and derived kernelled fuzzy point. By these notions, we obtain that the kernel of a set in fuzzy topological space (X, T) is a union of the set itself with the set of all boundary kernelled fuzzy points. In addition, it is a union of the set itself with the set of all derived kernelled fuzzy points and we give some result of FR_0- space by using these notions. Also in this paper we introduce fuzzy topological kr- space iff kernel of a subset A of X is an fuzzy open set. Via this kind of fuzzy topological space, we give a new characterization of fuzzy separation axioms lying between FT_0 and FT_1- spaces.

2. PRELIMINARIES

Fuzzy sets theory, introduced by Lotfi. A. Zada in 1965 [8], is the extension of classical set theory by allowing the membership of elements to range from 0 to 1. Let X be the universe of a classical set of objects. Membership in a classical subset A of X is often viewed as a characteristic function μ_A from X into $\{0, 1\}$, where

$$\mu_A(x) = \begin{cases} 1 & , \text{ for } x \in A \\ 0 & , \text{ for } x \notin A \end{cases} \quad \text{(see [4])}$$

for any $x \in X$.

$\{0,1\}$ is called a valuation set (see [13]). If the valuation set is allowed to be the real interval [0,1], A is called a fuzzy set in X. $\mu_A(x)$ (or simply $A(x)$) is the membership value (or degree of membership) of x in A. Clearly, A is a subset of X that has

no sharp boundary. A fuzzy set A in X can be represented by the set of pairs: $A = \{(x, A(x)), x \in X\}$. Let $A : X \to [0,1]$ be a fuzzy set. If $A(x) = 1$, for each $x \in X$, we denote it by 1_X and if $A(x) = 0$, for each $x \in X$, we denote it by 0_X . That is, by 0_X and 1_X , we mean the constant fuzzy sets taking the values 0 and 1 on X, respectively [2]. Let I =[0,1]. The set of all fuzzy sets in X, denoted by I^X [10].

Definition 2.1:[4] Let A be a fuzzy set of a set X. The support of A is the elements x whose membership value is greater than 0, i.e., supp(A) = $\{ x \in X : A(x) > 0\}$.

Definition 2.2:[5] Let A and B be any two fuzzy sets in X. Then we define $A \vee B : X \to [0,1]$ as follows:

$(A \vee B)(x) = max\ \{A(x), B(x)\}$. Also, we define $A \wedge B : X \to [0,1]$ as follows: $(A \wedge B)(x) = min\ \{A(x), B(x)\}$.

By $A \vee B$ ($A \wedge B$), we mean the union (intersection) between two fuzzy sets A and B of X.

Definition 2.3:[4] Let A be any fuzzy set in a set X. The complement of A, is denoted by $1_X - A$ or A^c and defined as follows: $A^c(x) = 1 - A(x)$, for each $x \in X$.

Remark 2.4: From definition (2.2) and definition (2.3) , we have, if $A,B \in I^X$, then $A \vee B$, $A \wedge B$ and $1_X - A \in I^X$.

Definition 2.5:[9] A fuzzy point x_λ in a set X is a fuzzy set defined as follows:

$$x_\lambda(y) = \begin{cases} \lambda & \text{if } y = x \\ 0 & \text{otherwise}, \end{cases}$$

Where $0 < \lambda \leq 1$. Now, supp(x_λ) = $\{ y : x_\lambda(y) > 0\}$, but

$$x_\lambda(y) = \begin{cases} \lambda & \text{if } y = x \\ 0 & \text{otherwise} \end{cases}, \quad \text{and } 0 < \lambda \leq 1 \text{ . Then,}$$

$\text{supp}(x_\lambda) = x$, so the value at x is λ, and call the point x its support of fuzzy point x_λ and λ is the height of x_λ. That is, x_λ has the membership degree 0 for all $y \in X$ except one, say $x \in X$.

Definition 2.6:[3] A fuzzy topology on a set X is a family T of fuzzy sets in X which satisfies the following conditions:
(i) 0_X , $1_X \in T$,
(ii) If A , $B \in T$, then $A \wedge B \in T$,
(iii) If $\{A_i : i \in J\}$ is a family in T, then $\vee_{i \in J} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T)(or simply X) is a fuzzy topological space or fts for short. Every element of T is called T - fuzzy open set (fuzzy open set, for short). A fuzzy set is T - fuzzy closed (or simply fuzzy closed), if its complement is fuzzy open set. As ordinary topologies, the indiscrete fuzzy topology on X contains only 0_X and 1_X (i.e., ϕ, X) , while the discrete fuzzy topology on X contains all fuzzy sets in X.

Example 2.7: Let $X = [-1,1]$, and let B_1 , B_2 and B_3 are fuzzy sets in X defined as follows:

$$B_1(x) = \begin{cases} 1 & \text{, if } -1 \leq x < 0 \\ 0 & \text{, if } 0 \leq x \leq 1 \end{cases},$$

$$B_2(x) = \begin{cases} 0 & \text{, if } -1 \leq x < 0 \\ 1 & \text{, if } 0 \leq x \leq 1 \end{cases},$$

$$B_3(x) = \begin{cases} 0 & \text{, if } -1 \leq x < 0 \\ 1/5 & \text{, if } 0 \leq x \leq 1 \end{cases}.$$

Let $T = \{ 0_X , B_1 , B_2 , B_3 , B_1 \vee B_3 , 1_X \}$, then T is a fuzzy topology on X, and (X, T) is a fts.

Example 2.8: Let $X = [0,1]$ and $T = \{0_X, K , 1_X \}$. Then T is a fuzzy topology on X, where $K : X \to [0,1]$ defined as:
$K(x) = x^2$, for all $x \in X$, and (X, T) is a fts.

Definition 2.9:[3] Let A be any fuzzy set in a fts X. The interior of A is the union of all fuzzy open sets contained in A, denoted by $int(A)$. That is, $int(A) = \vee\{B : B \text{ is fuzzy open set} , B \leq A\}$.

Definition 2.10:[3] Let A be any fuzzy set in a fts X. The closure of A is the intersection of all fuzzy closed sets containing A, denoted by $cl(A)$. That is, $cl(A) = \wedge\{B : B \text{ is fuzzy closed set}, B \geq A\}$.

The most important properties of the closure and interior of fuzzy sets are listed in the following proposition.

Proposition 2.11:[12] If A is any fuzzy set in X then:
(i) A is a fuzzy open (closed) set if and only if $A = int(A)$ ($A = cl(A)$),
(ii) $cl(1_X - A) = 1_X - int(A)$,
(iii) $int(1_X - A) = 1_X - cl(A)$.

Proposition 2.12:[13] Let A,B be two fuzzy sets in a fts X. Then:
(i) $int(A) \leq A$, $int(int(A)) = int(A)$,
(ii) $int(A) \leq int(B)$, whenever $A \leq B$,
(iii) $int(A \wedge B) = int(A) \wedge int(B)$, $int(A \vee B) \geq int(A) \vee int(B)$,
(iv) $A \leq cl(A)$, $cl(cl(A)) = cl(A)$,
(v) $cl(A) \leq cl(B)$, whenever $A \leq B$,
(vi) $cl(A \wedge B) \leq cl(A) \wedge cl(B)$, $cl(A \vee B) = cl(A) \vee cl(B)$.

Definition 2.13:[1] Let (X, T) be a fuzzy topological space. Then X is called:
(i) fuzzy T_0- space (FT_0- space, for short) iff for each pair of distinct fuzzy points in X, there exists an fuzzy open set in X containing one and not the other .
(ii) fuzzy T_1- space (FT_1- space, for short) iff for each pair of distinct fuzzy points x_λ and y_α of X, there exists an fuzzy open sets G, H containing x_λ and y_α respectively such that $y_\alpha \notin G$ and $x_\lambda \notin H$.
(iii) fuzzy T_2- space (FT_2- space, for short) iff for each pair of distinct fuzzy points x_λ and y_α of X, there exist disjoint fuzzy open sets G, H in X such that $x_\lambda \in G$ and $y_\alpha \in H$.
(iv) fuzzy regular space iff for each fuzzy closed set A and for each $x_\lambda \notin A$, there exist disjoint fuzzy open sets G, H such that $x_\lambda \in G$ and $A \leq H$.
(v) fuzzy normal space iff for each pair of disjoint fuzzy closed sets A and B, there exist disjoint fuzzy open sets G and H such that $A \leq G$ and $B \leq H$.

3. KERNEL SET IN FUZZY TOPOLOGICAL SPACES

Definition 3.1: The intersection of all fuzzy open subsets of a fuzzy topological space (X, T) containing A is called the kernel of A (briefly $ker(A)$), this means that $ker(A) = \wedge\{G \in T : A \leq G\}$.

Definition 3.2: In a fuzzy topological space (X, T), a set A is said to be weakly ultra separated from B if there exists a fuzzy open set G such that $A \leq G$ and $G \wedge B = 0_X$ or $A \wedge cl(B) = 0_X$.

Remark 3.3: By definition (3.2), we have the following: For every two distinct fuzzy points x_λ and y_α of X, $ker\{x_\lambda\} = \{ y_\alpha : \{x_\lambda\}$ is not weakly ultra separated from $\{ y_\alpha\} \}$.

Definition 3.4: A fuzzy topological space (X, T) is called fuzzy R_0- space (FR_0- space, for short) if for each fuzzy open set U and $x_\lambda \in U$ then $cl\{x_\lambda\} \leq U$.

Definition 3.5: A fuzzy topological space (X, T) is called fuzzy R_1- space (FR_1- space, for short) if for each two distinct fuzzy points x_λ and y_α of X with $cl\{x_\lambda\} \neq cl\{ y_\alpha\}$, there exist disjoint fuzzy open sets U, V such that $cl\{x_\lambda\} \leq U$ and $cl\{x_\lambda\} \leq V$.

Remark 3.6: Each fuzzy separation axiom is defined as the conjunction of two weaker axioms: FT_i- space $= FR_{i-1}$- space and FT_{i-1}- space $= FR_{i-1}$- space and FT_0- space, $i = 1,2$.

Lemma 3.7: Let (X,T) be a fuzzy topological space then $y_\alpha \in ker\{x_\lambda\}$ iff $x_\lambda \in cl\{ y_\alpha\}$, for each $x \neq y \in X$.

Proof: Let $y_\alpha \notin ker\{x_\lambda\}$, then there exist fuzzy open set V containing x_λ such that $y_\alpha \notin V$. Thus, $x_\lambda \notin cl\{ y_\alpha\}$.
The converse part can be proved in a similar way.

Theorem 3.8: A fuzzy topological space (X,T) is FT_1- space if and only if for each $x \neq y \in X$, $y_\alpha \notin ker\{x_\lambda\}$ and $x_\lambda \notin ker\{ y_\alpha\}$.

Proof: Let (X,T) be a FT_1- space then for each $x \neq y \in X$, there exists an fuzzy open sets U, V such that $x_\lambda \in U, y_\alpha \notin U$ and $y_\alpha \in V, x_\lambda \notin V$. Implies $y_\alpha \notin ker\{x_\lambda\}$ and $x_\lambda \notin ker\{ y_\alpha\}$.
Conversely, let $y_\alpha \notin ker\{x_\lambda\}$ and $x_\lambda \notin ker\{ y_\alpha\}$, for each $x \neq y \in X$. Then there exists an fuzzy open sets U, V such that
$x_\lambda \in U$, $y_\alpha \notin U$ and $y_\alpha \in V, x_\lambda \notin V$. Thus, (X,T) is a FT_1- space.

Theorem 3.9: A fuzzy topological space (X,T) is FT_1- space if and only if for each $x \in X$ then $ker\{x_\lambda\} = \{x_\lambda\}$.

Proof: Let (X,T) be a FT_1- space and let $ker\{x_\lambda\} \neq \{x_\lambda\}$, then $ker\{x_\lambda\}$ contains another fuzzy point distinct from x_λ say y_α.
So $y_\alpha \in ker\{x_\lambda\}$. Hence by theorem (3.8), (X,T) is not a FT_1- space this is a contradiction. Thus, $ker\{x_\lambda\} = \{x_\lambda\}$.
Conversely, let $ker\{x_\lambda\} = \{x_\lambda\}$, for each $x \in X$ and let (X,T) be not a FT_1- space. Then, by theorem (3.8) , $y_\alpha \in ker\{x_\lambda\}$, implies $ker\{x_\lambda\} \neq \{x_\lambda\}$, this is a contradiction. Thus, (X,T) is a FT_1- space.

Definition 3.10: Let (X,T) be a fuzzy topological space. A fuzzy point x_λ is said to be kernelled fuzzy point of $A \leq X$ (Briefly $x_\lambda \in ker(A)$) if and only if for each G fuzzy closed set contains x_λ then $G \wedge A \neq 0_X$.

Definition 3.11: Let (X,T) be a fuzzy topological space. A fuzzy point x_λ is said to be boundary kernelled fuzzy point of A (Briefly $x_\lambda \in kr_{bd}(A)$) if and only if for each fuzzy closed set G contains x_λ then $G \wedge A \neq 0_X$ and $G \wedge A^c \neq 0_X$.

Definition 3.12: Let (X,T) be a fuzzy topological space. A fuzzy point x_λ is said to be derived kernelled fuzzy point of A (Briefly $x_\lambda \in kr_{dr}(A)$) if and only if for each G fuzzy closed set contains x_λ then $A \wedge G \,/\, \{x_\lambda\} \neq 0_X$.

Definition 3.13: By definition (3.10), we have the following: For every two distinct fuzzy points x_λ and y_α of X, $ker\{x_\lambda\} = \{y_\alpha : x_\lambda \in G_{y_\alpha}, G^c_{y_\alpha} \in T\}$.
Theorem 3.14: Let (X,T) be a fuzzy topological space and $x \neq y \in X$. Then x_λ is a kernelled fuzzy point of $\{ y_\alpha\}$ if and only if y_α is an adherent fuzzy point of $\{x_\lambda\}$.

Proof: Let x_λ be a kernelled fuzzy point of $\{ y_\alpha\}$. Then for every fuzzy closed set G such that $x_\lambda \in G$ implies $y_\alpha \in G$, then
$y_\alpha \in \wedge \{G : x_\lambda \in G\}$, this means $y_\alpha \in cl\{x_\lambda\}$. Thus y_α is an adherent fuzzy point of $\{x_\lambda\}$.

Conversely, let y_α be an adherent fuzzy point of $\{x_\lambda\}$. Then for every fuzzy open set U such that $y_\alpha \in U$ implies $x_\lambda \in U$, then $x_\lambda \in \wedge\{U: y_\alpha \in U\}$, this means $x_\lambda \in ker\{y_\alpha\}$. Thus, x_λ is a kernelled fuzzy point of $\{y_\alpha\}$.

Theorem 3.15: Let (X,T) be a fuzzy topological space and $A \leq X$ and let $kr_{dr}(A)$ be the set of all kernelled derived fuzzy points of A, then $ker(A) = A \vee kr_{dr}(A)$.

Proof: Let $x_\lambda \in A \vee kr_{dr}(A)$ and if $x_\lambda \in kr_{dr}(A)$, then for every fuzzy closed set G intersects A (in a fuzzy point different from x_λ). Therefore, $x_\lambda \in ker\{x_\lambda\}$. Hence, $kr_{dr}(A) \leq ker(A)$, it follows that $A \vee kr_{dr}(A) \leq ker(A)$. To demonstrate the reverse inclusion, we consider x_λ be a fuzzy point of $ker(A)$. If $x_\lambda \in A$, then $x_\lambda \in A \leq kr_{dr}(A)$. Suppose that $x_\lambda \notin A$. Since $x_\lambda \in ker(A)$, then for every fuzzy closed set G containing x_λ implies $G \wedge A \neq 0_X$, this means $A \wedge G \,/\, \{x_\lambda\} \neq 0_X$. Then, $x_\lambda \in kr_{dr}(A)$, so that $x_\lambda \in A \vee kr_{dr}(A)$.

Theorem 3.16: Let (X,T) be a fuzzy topological space and $A \leq X$ and let $kr_{bd}(A)$ be the set of all kernelled boundary fuzzy points of A, then $ker(A) = A \vee kr_{bd}(A)$.

Proof: Let $x_\lambda \in A \vee kr_{bd}(A)$ and if $x_\lambda \in kr_{bd}(A)$, then for every fuzzy closed set G intersects A, therefore, $x_\lambda \in ker\{x_\lambda\}$. Hence, $kr_{bd}(A) \leq ker(A)$, it follows that $A \vee kr_{bd}(A) \leq ker(A)$. To demonstrate the reverse inclusion, we consider x_λ be a fuzzy point of $ker(A)$. If $x_\lambda \in A$, then $x_\lambda \in A \vee kr_{bd}(A)$. Suppose that $x_\lambda \notin A$, implies $x_\lambda \in A^c$. Since $x_\lambda \in ker(A)$,
then for every fuzzy closed set G containing x_λ implies $G \wedge A \neq 0_X$ and $G \wedge A^c \neq 0_X$. Then $x_\lambda \in kr_{bd}(A)$, so that $x_\lambda \in A \vee kr_{bd}(A)$. Hence, $ker(A) \leq A \vee kr_{bd}(A)$. Thus, $ker(A) = A \vee kr_{dr}(A)$.

Corollary 3.17: Every interior fuzzy point is a kernelled fuzzy point.

Proof: Since $int(A) \leq A \leq ker(A)$. Thus, every interior fuzzy point is a kernelled fuzzy point.

Theorem 3.18: Let (X,T) be a fuzzy topological space and A is a subset of X. Then A is a fuzzy open set if and only if every x_λ kernelled fuzzy point of A is an interior fuzzy point of A.

Proof: Let A be a fuzzy open set, then $ker(A) = A = int(A)$, implies every kernelled fuzzy point is an interior fuzzy point.
Conversely, let every x_λ kernelled fuzzy point of A is an interior fuzzy point of A. Then $ker(A) \leq int(A)$. Hence, $int(A) \leq A \leq ker(A)$, implies $int(A) = A = ker(A)$. Thus A is a fuzzy open set.

Corollary 3.19: A subset A of X is a fuzzy open set if and only if for each x_λ kernelled fuzzy point then $x_\lambda \notin cl(A^c)$.

Proof: By theorem (3.18).

Theorem 3.20: A subset A of X is a fuzzy closed set if and only if $ker(A^c) \wedge cl(A) = 0_X$.

Proof: Let A is a fuzzy closed set. Then A^c is a fuzzy open set, implies $A^c = ker(A^c)$ by theorem (3.18). Hence $A = cl(A)$. Thus $ker(A^c) \wedge cl(A) = 0_X$.
Conversely, let $ker(A^c) \wedge cl(A) = 0_X$, then for each $x_\lambda \in ker(A^c)$, implies $x_\lambda \notin cl(A)$, implies $x_\lambda \in ext(A)$. Therefore $x_\lambda \in int(A^c)$. Hence by theorem (3.18), A^c is a fuzzy open set. Thus A is a fuzzy closed set.

Theorem 3.21: A fuzzy topological space (X,T) is FR_0- space if and only if every adherent fuzzy point of $\{x_\lambda\}$ is a kernelled fuzzy point of $\{x_\lambda\}$.

Proof: Let (X,T) be an FR_0- space. Then, for each $x \in X, ker\{x_\lambda\} = cl\{x_\lambda\}$ by lemma (3.7). Thus, every adherent fuzzy point of $\{x_\lambda\}$ is a kernelled fuzzy point of $\{x_\lambda\}$.
Conversely, let every adherent fuzzy point of $\{x_\lambda\}$ is a kernelled fuzzy point of $\{x_\lambda\}$ and let $U \leq X$ and $x_\lambda \in U$. Then $cl\{x_\lambda\} \leq ker\{x_\lambda\}$ for each $x \in X$. Since $ker\{x_\lambda\} = \wedge\{U: U \in T, x_\lambda \in U\}$, implies $cl\{x_\lambda\} \leq U$, for each U fuzzy open set contains x_λ. Thus, (X,T) is an FR_0- space.

Theorem 3.22: A fuzzy topological space (X,T) is FT_0- space if and only if for each $x \neq y \in X$, either x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ or y_α is not kernelled fuzzy point of $\{x_\lambda\}$.

Proof: Let (X,T) be an FT_0- space. Then for each $x \neq y \in X$ there exists a fuzzy open set U such that $x_\lambda \in U$, $y_\alpha \notin U$ (say), implies $y_\alpha \in U^c$. Hence U^c is a fuzzy closed, then y_α is not kernelled fuzzy point of $\{x_\lambda\}$. Thus either x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ or y_α is not kernelled fuzzy point of $\{x_\lambda\}$.
Conversely, let for each $x \neq y \in X$, either x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ or y_α is not kernelled fuzzy point of $\{x_\lambda\}$. Then there exist a fuzzy closed set G such that $x_\lambda \in G$, $G \wedge \{ y_\alpha\} = 0_X$ or $y_\alpha \in G$, $G \wedge \{x_\lambda\} = 0_X$, implies $x_\lambda \notin G^c$, $y_\alpha \in G^c$ or $x_\lambda \in G^c$, $y_\alpha \notin G^c$. Hence G^c is a fuzzy open set. Thus, (X,T) is a FT_0- space.

Theorem 3.23: A fuzzy topological space (X,T) is FT_1- space if and only if $kr_{dr}\{x_\lambda\} = 0_X$, for each $x \in X$.

Proof: Let (X,T) be an FT_1- space. Then for each $x \in X, ker\{x_\lambda\} = \{x_\lambda\}$ by theorem (3.9). Since $kr_{dr}\{x_\lambda\} = ker\{x_\lambda\} - \{x_\lambda\}$. Thus, $kr_{dr}\{x_\lambda\} = 0_X$.
Conversely, let $kr_{dr}\{x_\lambda\} = 0_X$. By theorem (3.14), $ker\{x_\lambda\} = \{x_\lambda\} \vee kr_{dr}\{x_\lambda\}$, implies $ker\{x_\lambda\} = \{x_\lambda\}$. Hence, by theorem (3.9), (X,T) is a FT_1- space.

Theorem 3.24: A fuzzy topological space (X,T) is FT_1- space if and only if for each $x \neq y \in X$, x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ and y_α is not kernelled fuzzy point of $\{x_\lambda\}$.

Proof: Let (X,T) be a FT_1- space. Then for each $x \neq y \in X$ there exist fuzzy open sets U,V such that $x_\lambda \in U$, $y_\alpha \notin U$ and $y_\alpha \in V$, $x_\lambda \notin V$, implies $x_\lambda \in V^c$, $\{ y_\alpha\} \wedge V^c = 0_X$ and $y_\alpha \in U^c$, $\{x_\lambda\} \wedge U^c = 0_X$. Hence U^c and V^c are fuzzy closed sets. Thus x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ and y_α is not kernelled fuzzy point of $\{x_\lambda\}$.
Conversely, let for each $x \neq y \in X$, x_λ is not kernelled fuzzy point of $\{ y_\alpha\}$ and y_α is not kernelled fuzzy point of $\{x_\lambda\}$. Then, there exist fuzzy closed sets G_1, G_2 such that $x_\lambda \in G_1$, $G_1 \wedge \{ y_\alpha\} = 0_X$ and $y_\alpha \in G_2, G_2 \wedge \{x_\lambda\} = 0_X$, implies $x_\lambda \in G_2^c$, $y_\alpha \notin G_2^c$ and $y_\alpha \in G_1^c$, $x_\lambda \notin G_1^c$. Hence G_1^c and G_2^c are fuzzy open sets. Thus, (X,T) is FT_1- space.

4. *kr*- Spaces in Fuzzy Topological Spaces:

Definition 4.1: A fuzzy topological space (X,T) is said to be a kr- space if and only if for each subset A of X then, $ker(A)$ is a fuzzy open set.

Definition 4.2: A fuzzy topological kr- space (X,T) is called fuzzy T_k- space (FT_k- space, for short) if and only if for each $x_\lambda \in X$, then $kr_{dr}\{x_\lambda\}$ is a fuzzy open set.

Theorem 4.3: In fuzzy topological kr- space (X,T), every FT_1-space is a FT_k- space.

Proof: Let (X,T) be a FT_1- space. Then, for each $x_\lambda \in X$, $ker\{x_\lambda\} = \{x_\lambda\}$ by theorem (3.9). As $kr_{dr}\{x_\lambda\} = ker\{x_\lambda\} - \{x_\lambda\}$, implies $kr_{dr}\{x_\lambda\} = 0_X$. Thus, (X,T) is a FT_k- space.

Theorem 4.4: In fuzzy topological kr- space (X,T), every FT_k- space is a FT_0- space.

Proof: Let (X,T) be a FT_k- space and let $x \neq y \in X$. Then, $kr_{dr}\{x_\lambda\}$ is a fuzzy open set, therefore, there exist two cases:
(i) $y_\alpha \in kr_{dr}\{x_\lambda\}$ is a fuzzy open set. Since $x_\lambda \notin kr_{dr}\{x_\lambda\}$. Thus (X,T) is a FT_0- space.
(ii) $y_\alpha \notin kr_{dr}\{x_\lambda\}$, implies $y_\alpha \notin ker\{x_\lambda\}$. But $ker\{x_\lambda\}$ is a fuzzy open set. Thus (X,T) is a FT_0- space.

Definition 4.5: A fuzzy topological kr- space (X,T) is said to be fuzzy T_L- space (FT_L- space, for short) if and only if for each $x \neq y \in X$, $ker\{x_\lambda\} \wedge ker\{y_\alpha\}$ is degenerated (empty or singleton fuzzy set).

Theorem 4.6: In fuzzy topological kr- space (X,T), every FT_1- space is FT_L- space.

Proof: Let (X,T) be a FT_1- space. Then for each $x \neq y \in X$, $ker\{x_\lambda\} = \{x_\lambda\}$ and $ker\{y_\alpha\} = \{y_\alpha\}$ by theorem (3.9), implies $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = 0_X$. Thus (X,T) is a FT_L- space.

Theorem 4.7: In fuzzy topological kr- space (X,T), every FT_L- space is a FT_0- space.

Proof: Let (X,T) be a FT_L- space. Then for each $x \neq y \in X$, $ker\{x_\lambda\} \wedge ker\{y_\alpha\}$ is degenerated (empty or singleton fuzzy set). Therefore there exist three cases:
(i) $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = 0_X$, implies (X,T) is a FT_0- space.
(ii) $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = \{x_\lambda\}\ or\ \{y_\alpha\}$, implies $y_\alpha \notin ker\{x\}$ or $x_\lambda \notin ker\{y_\alpha\}$ implies (X,T) is a FT_0- space.
(iii) $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = \{z_\beta\}, z \neq x \neq y, z \in X$, implies $y_\alpha \notin ker\{x_\lambda\}$ and $x_\lambda \notin ker\{y_\alpha\}$, implies (X,T) is a FT_0- space.

Definition 4.8: A fuzzy topological kr- space (X,T) is said to be a fuzzy T_N- space (FT_N- space, for short) if and only if for each $x \neq y \in X$, $ker\{x_\lambda\} \wedge ker\{y_\alpha\}$ is empty or $\{x_\lambda\}$ or $\{y_\alpha\}$.

Theorem 4.9: In fuzzy topological kr- space (X,T), every FT_1- space is FT_N- space.

Proof: Let (X,T) be a FT_N- space. Then for each $x \neq y \in X$, $ker\{x_\lambda\} = \{x_\lambda\}$ and $ker\{y_\alpha\} = \{y_\alpha\}$ by theorem (3.9), implies $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = 0_X$. Thus (X,T) is a FT_N- space.

Theorem 4.10: In fuzzy topological kr- space (X,T), every FT_N- space is a FT_0- space.

Proof: Let (X,T) be a FT_N- space. Then for each $x \neq y \in X$, $ker\{x_\lambda\} \wedge ker\{y_\alpha\}$ is degenerated (empty or singleton fuzzy set). Therefore there exist two cases:
(i) $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = 0_X$, implies (X,T) is a FT_0- space.
(ii) $ker\{x_\lambda\} \wedge ker\{y_\alpha\} = \{x_\lambda\}$ or $\{y_\alpha\}$, implies $y_\alpha \notin ker\{x_\lambda\}$ or $x_\lambda \notin ker\{y_\alpha\}$, implies (X,T) is a FT_0- space.

Theorem 4.11: A fuzzy topological kr- space (X,T) is FT_2- space iff for each $x \neq y \in X$, then $ker\{x_\lambda\} \wedge ker\{ y_\alpha\} = 0_X$.

Proof: Let a fuzzy topological kr- space (X,T) is FT_2- space. Then for each $x \neq y \in X$ there exist disjoint fuzzy open sets U, V such that $x_\lambda \in U$, and $y_\alpha \in V$. Hence $ker\{x_\lambda\} \leq U$ and $ker\{ y_\alpha\} \leq V$. Thus $ker\{x_\lambda\} \wedge ker\{ y_\alpha\} = 0_X$.
Conversely, let for each $x \neq y \in X$, $ker\{x_\lambda\} \wedge ker\{ y_\alpha\} = 0_X$. Since (X,T) is a fuzzy topological kr- space, this means kernel is a fuzzy open set. Thus (X,T) is FT_2- space.

Theorem 4.12: A fuzzy topological kr- space (X,T) is fuzzy regular space iff for each G fuzzy closed set and $x_\lambda \notin G$, then $ker(G) \wedge ker\{x_\lambda\} = 0_X$.

Proof: By the same way of proof of theorem (4.11).

Theorem 4.13: A fuzzy topological kr- space (X,T) is fuzzy normal space iff for each disjoint fuzzy closed sets G, H, then $ker(G) \wedge ker(H) = 0_X$.

Proof: By the same way of proof of theorem (4.11).

Theorem 4.14: A fuzzy topological kr- space (X,T) is FT_1- space iff it is FR_0- space and FT_k-space.

Proof: It follows from theorem (4.3) and remark (3.6).
Theorem 4.15: A fuzzy topological kr- space (X,T) is FT_1- space iff it is FR_0- space and FT_L-space.

Proof: It follows from theorem (4.6) and remark (3.6).

Theorem 4.16: A fuzzy topological kr- space (X,T) is FT_1- space if and only if it is FR_0- space and FT_N- space.

Proof: It follows from theorem (4.9) and remark (3.6).

Theorem 4.17: A fuzzy topological kr- space (X,T) is FT_i- space if and only if it is FR_{i-1}- space and FT_k-space, $i = 1,2$.

Proof: It follows from theorem (4.3) and remark (3.6).

Theorem 4.18: A fuzzy topological kr- space (X,T) is FT_i- space if and only if it is FR_{i-1}- space and FT_L-space, $i = 1,2$.

Proof: It follows from theorem (4.6) and remark (3.6).

Theorem 4.19: A fuzzy topological kr- space (X,T) is FT_i- space if and only if it is FR_{i-1}- space and FT_N-space, $i = 1,2$.

Proof: It follows from theorem (4.9) and remark (3.6).

Remark 4.20: The relation between fuzzy separation axioms can be representing as a matrix. Therefore, the element a_{ij} refers to this relation. As the following matrix representation shows:

and	FT_0	FT_1	FT_2	FR_0	FR_1	FT_k	FT_L	FT_N
FT_0	FT_0	FT_1	FT_2	FT_1	FT_2	FT_k	FT_L	FT_N
FT_1	FT_1	FT_1	FT_2	FT_1	FT_2	FT_1	FT_1	FT_1
FT_2	FT_2	FT_2	FT_2	FT_2	FT_2	FT_2	FT_2	FT_2
FR_0	FT_1	FT_1	FT_2	FR_0	FR_1	FT_1	FT_1	FT_1
FR_1	FT_2	FT_2	FT_2	FR_1	FR_1	FT_2	FT_2	FT_2
FT_k	FT_k	FT_1	FT_2	FT_1	FT_2	FT_k	FT_L	FT_0
FT_L	FT_L	FT_1	FT_2	FT_1	FT_2	FT_L	FT_L	FT_0
FT_N	FT_N	FT_1	FT_2	FT_1	FT_2	FT_0	FT_0	FT_N

Matrix Representation

The relation between fuzzy separation axioms in fuzzy topological kr- spaces

REFERENCES

1. A. S. Mashour, E, E. Kerre and M. H. Ghanim, "Separation axioms, sub spaces and sums in fuzzy topology", J. Math Anal., 102(1984), 189-202.
2. B. Sikn, "On fuzzy FC - Compactness", Comm. Korean Math. Soc.13 (1998), 137-150.
3. C. L. Chang, "Fuzzy topological spaces", J. Math. Anal. Appl. 24(1968), 182-190.
4. G. J. Klir, U. S. Clair, B. Yuan, " Fuzzy set theory ", foundations and applications, 1997.
5. G. Balasubramanian, "Fuzzy β - open sets and fuzzy β - separation axioms", Kybernetika 35(1999), 215-223.
6. G. Balasubramania and P. Sundaram, "On some generalizations of fuzzy continuous functions", Fuzzy sets and Systems, 86(1) (1997), 93-100.
7. H. Maki et al., "Generalized closed sets in fuzzy topological space, I, Meetings on Topological spaces Theory and its Applications", (1998), 23-36.
8. L. A. Zadeh, "Fuzzy sets", Information, and control, 8 (1965), 338 -353.
9. M. Sarkar, "On fuzzy topological spaces", J. Math. Anal. Appl. 79 (1981), 384-394.
10. M. Alimohammady and M. Roohi, "On Fuzzy $\phi\psi$ - Continuous Multifunction", J. App. Math. sto. Anal. (2006), 1-7.
11. O. Berde Ozbaki, "On generalized fuzzy strongly semiclosed sets in fuzzy topological spaces", Int. J. Math. Sci. 30 (11) (2002), 651- 657.
12. R. H. Warren, "Neighborhoods, bases and continuity in fuzzy topological spaces", The Rocky Mountain Journal of Mathematics, Vol. 8, No.3, (1977), 459-470.
13. X. Tang, "Spatial object modeling in fuzzy topological spaces with application to lands cover change in china", Ph. D. Dissertation, ITC Dissertation No.108, Univ. of Twente, The Nether lands, (2004).
14. Gunwanti S. Mahajan and Kanchan S. Bhagat, "Medical Image Segmentation using Enhanced K-Means and Kernelized Fuzzy C- Means", International Journal of Electronics and Communication Engineering &Technology (IJECET), Volume 4, Issue 6, 2013, pp. 62 - 70, ISSN Print: 0976- 6464, ISSN Online: 0976 –6472.