Alpha Star Generalized ω - Closed Sets in Bitopological Spaces

By

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Abstract:

The aim of this paper is to introduce the concepts of alpha star generalized ω - closed sets, alpha star generalized ω - open sets and study their basic properties in bitopological spaces.

Keywords: $\tau_1\tau_2$ - alpha star generalized ω - closed sets, $\tau_1\tau_2$ - alpha star generalized ω - open sets, $\tau_1\tau_2$ - generalized ω - closed sets.

1. Introduction:

Levine, [7] initiated the study of generalized closed sets in topological spaces in 1970. In 1963, J. C. Kelly, [2] defined: a set equipped with two topologies is called a bitopological space, denoted by (X,τ_1,τ_2) where (X,τ_1) and (X,τ_2) are two topological spaces. Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by F. H. Khedr and H. S. Al-saadi, [1]. K. Chandrasekhara Rao and K. Kannan, [5,6] introduced the concepts of semi star generalized closed sets in bitopological spaces. Moreover, the concept of generalized closed sets were introduced in ideal bitopological spaces by Noiri and Rajesh [9]. In 1986, T. Fukutake, [8] generalized this notion to bitopological spaces and he defined a set A of a bitopological space X to be an *ij*-generalized closed set (briefly *ij*-*g*-closed) if $j\text{-}cl(A)\subseteq U$ whenever $A\subseteq U$ and U is τ_i-open in $X, i, j=1,2$ and $i\neq j$. For any subset $A\subseteq X$, $\tau_i\text{-int}(A)$ and $\tau_i\text{-}cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i, for $i=1,2$. The closure and interior with respect to the topology τ_i of B relative to A is written as $\tau_i\text{-}cl_B(A)$ and $\tau_i\text{-int}_B(A)$ respectively. A point $x\in X$ is called a condensation point of A if for each $U\in\tau$ with $x\in U$, the set $U\cap A$ is uncountable. A is called ω - closed if it contains all its condensation points. The complement of an ω - closed set is called ω - open. It is well known that a subset A of a space (X,τ) is ω - open if and only if for each $x\in A$, there exists $U\in\tau$ such that $x\in U$ and $U\cap W$ is countable. The family of all ω - open subsets of a space (X,τ), by τ_ω or $\omega O(X)$, forms a topology on X finer than τ. The ω - closure and ω - interior with respect to the topology τ_i, that can be defined in a manner similar to $\tau_i\text{-}cl(A)$ and $\tau_i\text{-int}(A)$, respectively, will be denoted by $\tau_i\text{-}cl_\omega(A)$ and $\tau_i\text{-int}_\omega(A)$, respectively. A^c or $X-A$ denotes the complement of A in X unless explicitly stated. The aim of this communication is to introduce the concepts of $\tau_1\tau_2$- alpha star generalized closed sets, $\tau_1\tau_2$- alpha star generalized ω - closed sets, $\tau_1\tau_2$- alpha star generalized ω - open sets and study their basic properties in bitopological spaces. We shall require the following known definitions.

Definition 1.1:

A subset A of a bitopological space (X,τ_1,τ_2) is called

(i) $\tau_1\tau_2$ - α -open [4] if $A \subseteq \tau_1\text{-int}(\tau_2\text{-}cl(\tau_1\text{-int}(A)))$.

(ii) $\tau_1\tau_2$ - α -closed [4] if $X-A$ is $\tau_1\tau_2$ - α -open.

Equivalently, a subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - α -closed if $\tau_2\text{-}cl(\tau_1\text{-int}(\tau_2\text{-}cl(A))) \subseteq A$.

(iii) $\tau_1\tau_2$ -generalized closed (briefly $\tau_1\tau_2$ - g -closed) [8] if $\tau_2\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open in X,

(iv) $\tau_1\tau_2$ -generalized open (briefly $\tau_1\tau_2$ - g -open) [8] if $X-A$ is $\tau_1\tau_2$ - g -closed.

(v) $\tau_1\tau_2$ - α generalized closed (briefly $\tau_1\tau_2$ - αg -closed) [4] if $\tau_2\text{-}\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open in X.

(vi) $\tau_1\tau_2$ - α generalized open (briefly $\tau_1\tau_2$ - αg -open) [4] if $X-A$ is $\tau_1\tau_2$ - αg -closed.

Definition 1.2:

A subset A of a bitopological space (X,τ_1,τ_2) is called

(i) $\tau_1\tau_2$ -generalized ω -closed (briefly $\tau_1\tau_2$ - $g\omega$ -closed) [3] if $\tau_2\text{-}cl_\omega(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open in X.

(ii) $\tau_1\tau_2$ -generalized ω -open (briefly $\tau_1\tau_2$ - $g\omega$ -open) [3] if $X-A$ is $\tau_1\tau_2$ - $g\omega$ -closed.

(iii) $\tau_1\tau_2$ - α generalized ω -closed (briefly $\tau_1\tau_2$ - $\alpha g\omega$ -closed) if $\tau_2\text{-}\alpha cl_\omega(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open in X.

(iv) $\tau_1\tau_2$ - α generalized ω -open (briefly $\tau_1\tau_2$ - $\alpha g\omega$ -open) if $X-A$ is $\tau_1\tau_2$ - $\alpha g\omega$ -closed.

2. Alpha Star Generalized Closed Sets:

In this section we define and study the concept of $\tau_1\tau_2$ - α^* generalized closed sets in bitopological spaces.

Definition 2.1:

A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - α^* generalized closed (briefly $\tau_1\tau_2$ - $\alpha^* g$ -closed) if $\tau_2\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 -open in X.

Example 2.2:

Let $X=\{a,b,c\}$, $\tau_1=\{\phi, X, \{a\}, \{b,c\}\}$, $\tau_2=\{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Then $\{a,b\}$ is $\tau_1\tau_2$ - $\alpha^* g$ -closed and $\{a\}$ is not $\tau_1\tau_2$ - $\alpha^* g$ -closed.

Definition 2.3:

A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ - α^* generalized open (briefly $\tau_1\tau_2$ - $\alpha^* g$ -open) if and only if $X-A$ is $\tau_1\tau_2$ - $\alpha^* g$ -closed.

Theorem 2.4:

The arbitrary union of $\tau_1\tau_2$ - $\alpha^* g$ - closed sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ - $\alpha^* g$ - closed if the family $\{A_i, i \in I\}$ is τ_2 - locally finite.

Proof:

Let $\{A_i, i \in I\}$ be τ_2 - locally finite and A_i is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X for each $i \in I$. Let $\bigcup A_i \subseteq U$ and U is τ_1 - open in X. Then, $A_i \subseteq U$ and U is τ_1 - open in X for each $i \in I$. Since A_i is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X for each $i \in I$, we have τ_2 - $cl(A_i) \subseteq U$. Consequently, $\bigcup[\tau_2\text{-}cl(A_i)] \subseteq U$. Since the family $\{A_i, i \in I\}$ be τ_2 - locally finite, $\tau_2\text{-}cl(\bigcup(A_i)) = \bigcup(\tau_2\text{-}cl(A_i)) \subseteq U$. Therefore, $\bigcup A_i$ is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X. ■

Theorem 2.5:

The arbitrary intersection of $\tau_1\tau_2$ - $\alpha^* g$ - open sets $A_i, i \in I$ in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ - $\alpha^* g$ - open if the family $\{A_i^c, i \in I\}$ is τ_2 - locally finite.

Proof:

Let $\{A_i^c, i \in I\}$ be τ_2 - locally finite and A_i is $\tau_1\tau_2$ - $\alpha^* g$ - open in X for each $i \in I$. Then, A_i^c is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X for each $i \in I$. Then by theorem (2.4), we have $\bigcup(A_i^c)$ is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X. Consequently, $(\bigcap A_i)^c$ is $\tau_1\tau_2$ - $\alpha^* g$ - closed in X. Therefore, $\bigcap A_i$ is $\tau_1\tau_2$ - $\alpha^* g$ - open in X. ■

3. Alpha Star Generalized ω - Closed Sets:

In this section we define and study the concept of $\tau_1\tau_2$ - α^* generalized ω - closed sets in bitopological spaces.

Definition 3.1:

A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ - α^* generalized ω - closed (briefly $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed) if $\tau_2\text{-}cl_\omega(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 - open in X.

Example 3.2:

Let X be the set of all real numbers R, $\tau_1 = \{\phi, R, R-Q\}$, $\tau_2 = \{\phi, R, Q\}$, where Q is the set of all rational numbers. Then $R-Q$ is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed.

Theorem 3.3:

Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.

(i) If A is τ_2 - ω - closed, then A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed.

(ii) If A is τ_1 - open and $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, then A is τ_2 - ω - closed.

(iii) If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, then A is $\tau_1\tau_2$ - $g\omega$ - closed.

Proof:

(i) Suppose that A is τ_2 - ω - closed, let $A \subseteq U$ and U is τ_1 - open in X. Then $\tau_2\text{-}cl_\omega(A) = A \subseteq U$. Consequently, A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed.

(ii) Suppose that A is τ_1 - open and $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed. Let $A \subseteq A$ and A is τ_1 - open. Then $\tau_2\text{-}cl_\omega(A) \subseteq A$. Therefore,

$\tau_2\text{-}cl_\omega(A) = A$. Consequently A is $\tau_2\text{-}\omega\text{-}$closed.

(iii) Suppose that A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed, let $A \subseteq U$ and U is τ_1-open in X. Since U is τ_1-open in X, we have $\tau_2\text{-}cl_\omega(A) \subseteq U$. Consequently, A is $\tau_1\tau_2\text{-}g\omega$-closed. ■

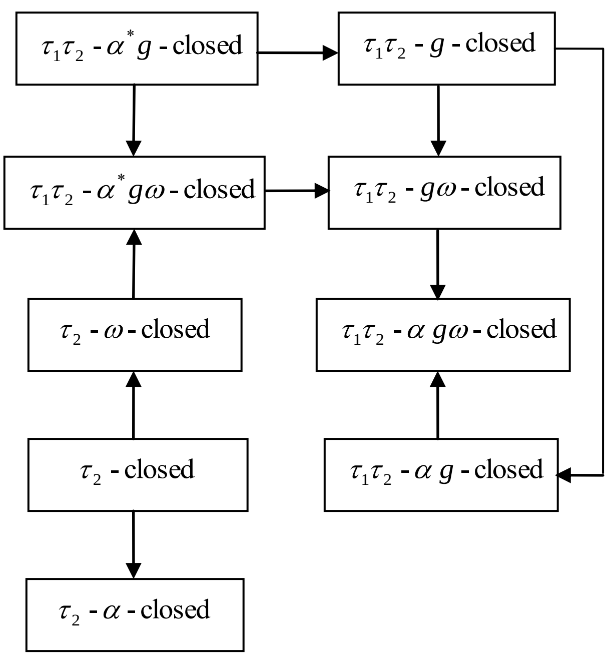
Since, $\tau_2\text{-}cl_\omega(A) \subseteq \tau_2\text{-}cl(A)$, we have the following theorem.

Theorem 3.4:

Every $\tau_1\tau_2\text{-}\alpha^* g$-closed set is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed and every τ_2-closed set is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed.

Remark 3.5:

From the theorem (3.3), theorem (3.4) and above definitions, we have the following relations.



Theorem 3.6:

If A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed in X and $A \subseteq B \subseteq \tau_2\text{-}cl_\omega(A)$, then B is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed.

Proof:

Suppose that A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed in X and $A \subseteq B \subseteq \tau_2\text{-}cl_\omega(A)$. Let $B \subseteq U$ and U is τ_1-open in X. Then $A \subseteq U$. Since A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed, we have $\tau_2\text{-}cl_\omega(A) \subseteq U$. Since $B \subseteq \tau_2\text{-}cl_\omega(A)$, $\tau_2\text{-}cl_\omega(B) \subseteq \tau_2\text{-}cl_\omega(A) \subseteq U$. Hence B is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed. ■

Theorem 3.7:

If A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed sets then so is $A \cup B$.

Proof:

Suppose that A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed sets . Let U be τ_1-open in X and $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed sets, we have $\tau_2\text{-}cl_\omega(A) \subseteq U$ and $\tau_2\text{-}cl_\omega(B) \subseteq U$. Consequently, $\tau_2\text{-}cl_\omega(A \cup B) \subseteq U$. Therefore, $A \cup B$ is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed. ■

Theorem 3.8:

Let $B \subseteq A \subseteq X$ where A is τ_1-open and $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed in X. Then B is $\tau_1\tau_2\text{-}\alpha^* g\omega$- closed relative to A if and only if B is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed relative to X.

Proof:

Suppose that $B \subseteq A \subseteq X$ where A is τ_1-open and $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed. Suppose that B is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed relative to A. Let $B \subseteq U$ and U is τ_1-open in X. Since

$A \subseteq X$, A is τ_1 - open , we have $A \cap U$ is τ_1 - open in X . Consequently $A \cap U$ is τ_1 - open in A .Since $B \subseteq A$, $B \subseteq U$, we have $B \subseteq A \cap U$. Since B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed relative to A, we have τ_2 - $cl_\omega(B_A)$ $\subseteq A \cap U$. Therefore , τ_2 - $cl_\omega(B_A) \subseteq U$. Since A is τ_1 - open , we have τ_2 - $cl_\omega(B_A) =$ τ_2 - $cl_\omega(B) \cap A = \tau_2$ - $cl_\omega(B) \subseteq U$. Hence B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed relative to X.

Conversely, suppose that B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed relative to X . Let $B \subseteq U$ and U is τ_1 - open in A . Since $A \subseteq X$, we have U is τ_1 - open in X . Since B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed relative to X, we have τ_2 - $cl_\omega(B) \subseteq U$. Now , τ_2 - $cl_\omega(B_A) =$ τ_2 - $cl_\omega(B) \cap A = \tau_2$ - $cl_\omega(B) \subseteq U$. Therefore , B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed relative to A. ■

Corollary 3.9:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, τ_1 - open in X and F is τ_2 - ω - closed in X , then $A \cap F$ is τ_2 - ω - closed in X.

Proof:

Since A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed , τ_1 - open in X , we have A is τ_2 - ω - closed in X . { By Theorem (3.3) (ii) }. Since F is τ_2 - ω - closed in X , $A \cap F$ is τ_2 - ω - closed in X. ■

Theorem 3.10:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X, then τ_2 - $cl_\omega(A) - A$ contains no nonempty τ_1 - closed set .

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X. Let F be τ_1 - closed and $F \subseteq \tau_2$ - $cl_\omega(A) - A$. Since F be τ_1 - closed , we have F^c is τ_1 - open .

Since $F \subseteq \tau_2$ - $cl_\omega(A) - A$, we have $F \subseteq \tau_2$ - $cl_\omega(A)$ and $A \subseteq F^c$. Since A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X, we have τ_2 - $cl_\omega(A) \subseteq F^c$. Consequently, $F = \phi$. Hence τ_2 - $cl_\omega(A) - A$ contains no nonempty τ_1 - closed set. ■

Corollary 3.11:

Let A be $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, then A is τ_2 - ω - closed if and only if τ_2 - $cl_\omega(A) - A$ is τ_1 - closed .

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed. Since A is τ_2 - ω - closed, we have τ_2 - $cl_\omega(A) = A$. Then τ_2 - $cl_\omega(A) - A = \phi$ is τ_1 - closed . Conversely, suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed and τ_2 - $cl_\omega(A) - A$ is τ_1 - closed . Since A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, we have τ_2 - $cl_\omega(A) - A$ contains no nonempty τ_1 - closed set { by Theorem (3.10) }. Since τ_2 - $cl_\omega(A) - A$ is itself τ_1 - closed , we have τ_2 - $cl_\omega(A) - A = \phi$. Then τ_2 - $cl_\omega(A) = A$. Hence A is τ_2 - ω - closed. ■

Theorem 3.12:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed and $A \subseteq B \subseteq \tau_2$ - $cl_\omega(A)$, then τ_2 - $cl_\omega(B) - B$ contains no nonempty τ_1 - closed set.

Proof:

Let A be $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed and

$A \subseteq B \subseteq \tau_2\text{-}cl_\omega(A)$. Then B is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed.{By Theorem (3.6)}. Hence $\tau_2\text{-}cl_\omega(B) - B$ contains no nonempty τ_1-closed set.{By Theorem (3.10)}. ■

4. Alpha Star Generalized ω - Open Sets:

We begin this section with a relatively new definition.

Definition 4.1:

A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2\text{-}\alpha^*$ generalized ω-open (briefly $\tau_1\tau_2\text{-}\alpha^* g\omega$-open) if and only if $X - A$ is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed.

Example 4.2:

In Example (3.2), Q is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open.

Theorem 4.3:

A set A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open if and only if $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$ whenever F is τ_1-closed and $F \subseteq A$.

Proof:

Suppose that A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open. Suppose that F is τ_1-closed and $F \subseteq A$. Then F^c is τ_1-open and $A^c \subseteq F^c$. Since A^c is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed, we have $\tau_2\text{-}cl_\omega(A^c) \subseteq F^c$. Since $\tau_2\text{-}cl_\omega(A^c) = (\tau_2\text{-}\mathrm{int}_\omega(A))^c$, we have $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$. Conversely, suppose that $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$ whenever F is τ_1-closed and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is τ_1-open. Since $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$, and $\tau_2\text{-}cl_\omega(A^c) = (\tau_2\text{-}\mathrm{int}_\omega(A))^c$, we have $\tau_2\text{-}cl_\omega(A^c) \subseteq U$. Then A^c is $\tau_1\tau_2\text{-}\alpha^* g\omega$-closed. Consequently, A is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open. ■

Theorem 4.4:

If A and B are separated $\tau_1\tau_2\text{-}\alpha^* g\omega$-open sets then $A \cup B$ is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open set.

Proof:

Suppose A and B are separated $\tau_1\tau_2\text{-}\alpha^* g\omega$-open sets. Let F be τ_1-closed and $F \subseteq A \cup B$. Since A and B are separated, we have $\tau_1\text{-}cl(A) \cap B = A \cap \tau_1\text{-}cl(B) = \phi$ and $\tau_2\text{-}cl(A) \cap B = A \cap \tau_2\text{-}cl(B) = \phi$. Then, $F \cap \tau_2\text{-}cl(A) \subseteq (A \cup B) \cap \tau_2\text{-}cl(A) = A$. Similarly, we can prove $F \cap \tau_2\text{-}cl(B) \subseteq B$. Since F is τ_1-closed, we have $F \cap \tau_1\text{-}cl(A)$ and $F \cap \tau_1\text{-}cl(B)$ are τ_1-closed. Since A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-open, we have $F \cap \tau_2\text{-}cl(A) \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$ and $F \cap \tau_2\text{-}cl(B) \subseteq \tau_2\text{-}\mathrm{int}_\omega(B)$. Now, $F = F \cap (A \cup B) \subseteq [F \cap \tau_2\text{-}cl(A)] \cup [F \cap \tau_2\text{-}cl(B)] \subseteq \tau_2\text{-}\mathrm{int}_\omega(A \cup B)$. Therefore, $A \cup B$ is $\tau_1\tau_2\text{-}\alpha^* g\omega$-open. ■

Theorem 4.5:

If A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-open sets then so is $A \cap B$.

Proof:

Suppose that A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-open sets. Let F be τ_1-closed and $F \subseteq A \cap B$. Then, $F \subseteq A$ and $F \subseteq B$. Since A and B are $\tau_1\tau_2\text{-}\alpha^* g\omega$-open, we have $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A)$ and $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(B)$. Hence $F \subseteq \tau_2\text{-}\mathrm{int}_\omega(A \cap B)$.

Consequently, $A \cap B$ is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open set. ■

Theorem 4.6:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open in X and τ_2 - $\text{int}_\omega(A) \subseteq B \subseteq A$, then B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open.

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open in X and τ_2 - $\text{int}_\omega(A) \subseteq B \subseteq A$. Let F be τ_1 - closed and $F \subseteq B$. Since $F \subseteq B$, $B \subseteq A$, we have $F \subseteq A$. Since A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open, we have $F \subseteq \tau_2$ - $\text{int}_\omega(A)$. Since τ_2 - $\text{int}_\omega(A) \subseteq B$, we have τ_2 - $\text{int}_\omega(A) \subseteq \tau_2$ - $\text{int}_\omega(B)$. Then $F \subseteq \tau_2$ - $\text{int}_\omega(B)$. Therefore , B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open set. ■

Theorem 4.7:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X then τ_2 - $cl_\omega(A) - A$ is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open.

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X. Let F be τ_1 - closed and $F \subseteq \tau_2$ - $cl_\omega(A) - A$. Since A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed in X, τ_2 - $cl_\omega(A) - A$ contains no nonempty τ_1 - closed set. Since $F \subseteq \tau_2$ - $cl_\omega(A) - A$, $F = \phi \subseteq \tau_2$ - $\text{int}_\omega(\tau_2$ - $cl_\omega(A) - A)$. Therefore, τ_2 - $cl_\omega(A) - A$ is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open. ■

Theorem 4.8:

If A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open in a bitopological space (X,τ_1,τ_2), then $G = X$ whenever G is τ_1 - open and τ_2 - $cl_\omega(A) \cup A^c \subseteq G$.

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open in a bitopological space (X,τ_1,τ_2) and G is τ_1 - open and τ_2 - $cl_\omega(A) \cup A^c \subseteq G$. Then, $G^c \subseteq (\tau_2$ - $\text{int}_\omega(A) \cup A^c)^c$ $= \tau_2$ - $cl_\omega(A^c) - A^c$. Since G^c is τ_1 - closed and A^c is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed, we have τ_2 - $cl_\omega(A^c) - A^c$ contains no nonempty τ_1 - closed set in X {By Theorem (3.10)}. Therefore, $G^c = \phi$. Hence $G = X$. ■

Theorem 4.9:

The intersection of a $\tau_1\tau_2$ - $\alpha^* g\omega$ - open set and τ_1 - ω - open set is always $\tau_1\tau_2$ - $\alpha^* g\omega$ - open.

Proof:

Suppose that A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open and B is τ_1 - ω - open. Then B^c is τ_2 - ω - closed. Therefore, B^c is $\tau_1\tau_2$ - $\alpha^* g\omega$ - closed. {By Theorem (3.3) (i)}. Hence B is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open. Consequently, $A \cap B$ is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open.{By Theorem (4.5)}. ■

Theorem 4.10:

If $A \times B$ is $\tau_1 \times \sigma_1\, \tau_2 \times \sigma_2$ - $\alpha^* g\omega$ - open subset of $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$, then A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open subset in (X,τ_1,τ_2) and B is $\sigma_1\sigma_2$ - $\alpha^* g\omega$ - open subset in (Y,σ_1,σ_2).

Proof:

Let F be a τ_1 - closed subset of (X,τ_1,τ_2) and let G be a σ_1 - closed subset of (Y,σ_1,σ_2) such that $F \subseteq A$ and $G \subseteq B$. Then $F \times G$ is $\tau_1 \times \sigma_1$ - closed in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ such that $F \times G \subseteq A \times B$. By assumption $A \times B$ is $\tau_1 \times \sigma_1\ \tau_2 \times \sigma_2$ - $\alpha^* g\omega$ - open in $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$ and so $F \times G \subseteq$ $\tau_2 \times \sigma_2$ - $\text{int}_\omega(A \times B) \subseteq \tau_2$ - $\text{int}_\omega(A) \times$ σ_2 - $\text{int}_\omega(B)$. Therefore $F \subseteq \tau_2$ - $\text{int}_\omega(A)$ and $G \subseteq \sigma_2$ - $\text{int}_\omega(B)$. Hence A is $\tau_1\tau_2$ - $\alpha^* g\omega$ - open and B is $\sigma_1\sigma_2$ - $\alpha^* g\omega$ - open. ■

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المستخلص:

الهدف من هذا البحث هو تقديم مفاهيم مجموعات الفا ستار المعممة ω - المغلقة ، مجموعات الفا ستار المعممة ω - المفتوحة ودراسة خصائصها الأساسية في الفضاءات ثنائية التبولوجي.