INTERNATIONAL JOURNAL OF ADVANCED RESEARCH IN ENGINEERING AND TECHNOLOGY (IJARET)

ISSN 0976 - 6480 (Print)
ISSN 0976 - 6499 (Online)
Volume 5, Issue 4, April (2014), pp. 27-30
© IAEME: www.iaeme.com/ijaret.asp
Journal Impact Factor (2014): 7.8273 (Calculated by GISI)
www.jifactor.com

IJARET
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ON THE EXPENSIVE FUNCTION AND CHAOTIC GROUP ACTIONS

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ABSTRACT

If (M, G, f) is topological transformation group, then it's called expensive if $\exists \delta > 0$ such that $\forall p, q \in M, p \neq q$. There exists t such that $d[f(p,t), f(q,t)] \geq \delta$.Every expensive function possess sensitive dependence on initial condition on X. Hence every expensive function is chaotic on X .[1]A chaotic group action in this paper is a finite group, where chaotic functions $F\ g\ \times\ f$, such as group study. If f is a cyclic group, specifically, a topological space which, under suitable assumptions $Z\ \times\ f$ is a homeomorphism that a chaotic action must admit, which treats a chaotic group action But a topological space that is a chaotic homeomorphism is implemented.

In this research we develop the theory of chaotic group operations. our research although its most common setting in place of a verb as a dynamical system in the most chaotic denned, was concerned that the dynamics of single maps (discrete dynamical systems); Inspired by this fact with the iteration in other words, the additive group Z Group action or semi with Z^+monotheism group we non-compact groups and how to construct a minimal transitive, and recurrent show G-actions.[3]

Keywords: Chaotic Actions; Topological Transitivity; Residually Finite Groups;

1. INTRODUCTION

The following assumption was a chaotic group of action launched a group, g, M; no action is taken on a space, chaotic, if it is topologically transitive, with a dense subset of points and finite set of classes. In this paper we compare compact triangularly manifold dimension of a faithful chaotic Action admits that more than suggests that a group assume that definition.., on a Hausdorff topological space m consecutive functions. Then we say that g if the following two conditions met the action M.

(a) Topological transitivity to each pair of chaotic: empty open subsets of M and V There is an element $g, \hat{E}$, such that $(U) \cap V \neq ø$.[2]

(b) A dense subset of dense finite classes: m G under which class is the set of points in finite.

This definition generalizes the definition of chaotic maps directly to Delaney. In particular, the initial conditions are chaotic functions on metric spaces infinite sensitive dependency. Notice that the above definition, just an abstract group g; we do not assume any particular topology on. "in this context is a constant battle just one action by homeomorphisms, $'M$. such action everything was taken by some elements of m g if loyal; That is, if the identity element of the point stabilizer subgroup M of g is residually finite element is that if and only if it is a group of loyal chaotic action was shown in some place close. While every compact surface admits a chaotic Z-action that in any group of circle a chaotic action.[3]

Chaotic dynamical systems have been the subject of intensive research in current state. In this thesis we develop the theory of chaotic group actions. An increasingly popular definition of chaos is due to Devaney. A map f is said to be chaotic on a metric space X if it satisfies the following conditions:

1) f has sensitive dependence on initial conditions; that is, there exists $\delta > 0$ such that for any $x \in X$ and any open neighbourhood $U\, of\, x$, there exists $y \in U$ and $n \geq 0$ such that $d\,(f^n(x), f^n(y)) > \delta,$

2) fIs topologicallytransitive; that is, for any pair of non-empty open disjoint sets U and V there exists n > 0 such $that\ f^n(U) \cap V \neq \emptyset,$

3) f Has a dense set of periodic points; that is the set $\{x\ X: f^n(x)x\ for\ some\ n \in N\}$ is dense in X.[9]

These conditions are respectively to reflect "an element of uncertainty, in decomposability and regularity" a chaotic system. It more recently has shown that topological transitivity and periodic classes of a set of thick sensitive dependency understood in any metric space.

As a natural generalization of Delaney's definition of chaotic maps, we introduce in this thesis the notion of chaotic action of arbitrary groups: an action is chaotic if it is topologically transitive and the set of points with finite orbits is dense. This notion is not merely an artificial generalization of Delaney's definition, since there exist chaotic actions of a group G for which the restriction to everyone generator subgroup is not chaotic.[5]

There are some questions which immediately come to mind when one familiarizes oneself with the definition of chaotic group action: which groups can have interesting chaotic dynamics on some Hausdorff space; which spaces admit chaotic actions of some group? We give answers to both questions. In answering the first question we show that a group G possesses a faithful chaotic action on some Hausdorff space if and only if G is residually finite. In fact, the density of finite orbits of a chaotic action, the condition in the definition of chaos which reflects "an element of regularity" of a chaotic system, is just a disguised form of residual finiteness. This result is especially interesting since residually finite groups are a well studied class of groups; they often occur in geometrical problems and have interesting group-theoretical properties. We remark in passing that the above result also provides an elementary and unified method to prove residual finiteness of groups in many cases.

In topological dynamics there are two slightly different approaches to studying group actions. One approach is to study an action of a group G by homeomorphisms on a topological space M; that is, the action defined by the homomorphism $\emptyset: G -> Homeo\ (M)$. Another approach is to study an action of a transformation group $G\ on\ M$; that is the action of a topological group G defined by the homomorphism $Ø: G —> Homeo\ (M)$ with the additional assumptions that the map $f: G\ x\ M —> M$ is continuous. Clearly, a group G acting on a space M by homeomorphisms can be made into a transformation group by equipping it with the discrete topology. In our study we adopt the first, more general, and approach. So, we do not usually assume any topological hypothesis on the groups. However we make one exception, to study a natural Hausdorff topology associated with residually finite groups, the profinite topology. We show that when a group G is given the profinite topology and the action of G is chaotic on a complete metric space M then the function $f: G\ x\ M —> M$ is discontinuous; that is, G is not a transformation group.[3],[4]

2. SCOPE AND OBJECTIVES OF THE STUDY

A sensitive dependence on initial conditions compact metric space is a continuous action of a group g if there is a number $e > 0$ that sets we can find g in g such that g greater diameter than e. U do U have to open full support if a such we live action is a measure of the likely preserves, The system either proves that at least and equicontinuous, or is sensitive dependence on initial conditions Glasner and generalizes the theorem invertible case Weiss. When we have finitely generated solvable groups, acts and is set to the minimum score of cyclical sub actions dense then the system has proven sensitive dependence on initial conditions. In addition, we non-transitive, non-compact monotheism groups, and very low, almost-equicontinuous, recurrent, group tasks show how to build example.[6]

3. CONSTRUCTION OF CHAOTIC GROUP ACTIONS

First recall that there are many examples of chaotic Z-actions; that is, chaotic homorphisms. Perhaps the basic example is that of the Anson diffeomorphisms of tore and infantile manifolds; these maps are chaotic since their periodic points are dense and by Anoso's closing lemma, they are transitive on their no wandering set. Let us now give some example-
Theorem: Consider a Hausdorff space M and the group Hom (M) of homeomorphisms of M. Then one has:

a) If there are group includes $G \leq H \leq K \leq Hom\ (M)$
Then the action of H on M is chaotic if the actions of G and K on M are chaotic.

b) If $G \leq H \leq Hom\ (M)$ and G has finite index in H and if the action of G on M is chaotic, then the action of H is chaotic.

c) If M is locally compact and if $Hom(M)$ is given the compact-open topology, then the action of $G\ on\ M$ is chaotic if and only if the action on M of the closure $G'of\ G\ in\ Hom(M)$ is chaotic.[7]

Proof: In Part (a), notice that if a point x Є M has finite orbit under K, then x obviously has finite orbit under H. So if the action of K has finite orbit dense, then the action of H has finite orbits dense. On the other hand, if the action of G is topologically transitive, then clearly the action of H is also topologically transitive. So part (a) holds. Part (b) is similar Part (a).

In Part©, again if the action of G' has finite orbits dense, then the action of G has finite orbits dense. Now suppose that the action of G' is topologically transitive. Let U and V be two non-empty open subsets of M. Then there exists g Є G' such that g (U) ∩ V is non-empty. Let x be an element of U ∩ $g^{-1(V)}$ and let Ø be the open subset of G' composed of elements that send x into V. Then g Є Ø and since G is dense in G', there exists h Є G ∩ Ø. So h (U) ∩ V is non empty and hence the action of G is topologically transitive.

Conversely, if M is locally compact, then the natural map Hom (M) * M —> M is continuous. So, if a point x Є M has finite orbit under G, then since G is dense in G', one has that G(x) is dense in G'(x). Hence G'(x) is finite. So if the action of the action of g is topologically transitive, then obviously so too is the action of G'.[8]

4. CHAOTIC ACTIONS OF TOPOLOGICAL SEMI GROUPS

This topology topological semi groups and sustainable actions proposed in the context of a general impression of chaos is to explore and discover that any chaotic actions one sensitive to initial conditions on the Hausdorff uniform space.

Chaos theory is of dynamical systems s.t core phenomena udied. Chaos Delaney well recognized definition of the concept of sensitive dependence on initial conditions and hence the metric, or at least considered the phase space uniform respectively. However, banks etc., which have been modified and enlarged in various ways, by a well known result suggests that the notion of chaos in terms of the topology to be completely renovated.

Let S be a semi group and let T ⊆ S. Concerning an element $s \subseteq S, let\ s^{-1}T: \{x \in S \mid sx \in T\}$ and $Ts^{-1} := \{x \in S \mid xs \in T\}$.
$For\ a\ subset\ K \subseteq S, define\ K^{-1}T: = Us \in K\ s^{-1}T\ and\ T\ K^{-1} := Us \in kTs^{-1}$.[2]

5. CONCLUSION

The aim of the present paper is to generalize Delaney's notion of chaos for the comprehensive context of topological semi groups in That remains valid and results refer to a discrete chaotic system or real-time with classic case is the most natural way as a particular instance is such a way that arises. Since our aim of a comparably comprehensive settings is some generalizations and try to find the appropriate chaos. Among other things, we have a fairly suggestive for our concept of chaos give an example Hausdorff uniform step spacing on continuous semi group actions to provide a generalization of results of banks.

REFERENCE

[1] Gottschalk. W.H; and Hedlund, G.A; Topological Dynamics, American Mathematical Society Colloquium Publications. Vol. 36 Am. Math. So., Providence, R. I. ,1955.

[2] J. Banks, Regular periodic decompositions for topologically transitive maps, Ergodic Theory Dynamical Systems 17 (1997) 505–529.

[3] G. Cairns, G. Davis, E. Elton, A. Kolganova, P. Perverse, Chaotic group actions, Ensign. Math. 41 (1995) 123–133.

[4] G. Cairns, A. Kolganova, Chaotic actions of free groups, Nonlinearity 9 (1996) 1015–1021.

[5] H. Furstenberg, Disjointness in ergodic theory, minimal sets, and a problem in Diophantine approximation, Math. Systems Theory 1 (1967) 1–49.

[6] D. Montgomery, L. Zipping, Topological Transformation Groups, Inter science, New York, 1955.

[7] P. Sankaran, K. Varadarajan, On certain homeomorphism groups, J. Pure Appl. Algebra 92 (1994) 191–197; Corrections, J. Pure

[8] R.L. Devaney, Linked twist mappings are almost Anson, Lecture Notes in Math., Vol. 819, Springer, Berlin, 1980, pp. 121–145.

[9] J. Banks, J. Brooks, G. Cairns, G. Davis, and P. Stacey, On Delaney's definition of chaos, Am. Math. Mon. 99 (1992), no. 4, 332-334.